Hale School

Mathematics Specialist

Term 1 2019

Test 2 - Functions

Name: SOLUTIONS

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Instructions:

- **Classpad and scientific calculators are not allowed**
- **External notes are not allowed**
- **Duration of test: 40 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (otherwise, no marks awarded)**
- **This test contributes to 6% of the year (school) mark**

Question 1 (9 marks)

Consider the functions $f(x) = \dfrac{9}{x^2}$ and $g(x) = \sqrt{1-x}$

(a) Find

i) $g \circ f(x)$ (1 mark)

$$= \sqrt{1 - \dfrac{9}{x^2}}$$

✓ answer

ii) the natural domain for $g \circ f(x)$ (2 marks)

Need $x \neq 0$ and $\dfrac{9}{x^2} \leq 1$

$\therefore x^2 \geq 9$

$D_{gf} = \{x : x \leq -3\} \cup \{x : x \geq 3\}$

✓ finds $\pm\sqrt{3}$

✓ correct domains

iii) the range for $g \circ f(x)$ corresponding to the domain in part ii) (2 marks)

$$R_{gf} = \{y : 0 \leq y < 1\}$$

✓ boundaries

✓ inequality signs

(b) State a domain for $g \circ f(x)$ such that it is a one-to-one function. (1 mark)

Restricted domain $D_{gf} = \{x : x \geq 3\}$

✓ answer

(c) For the domain in part (b), find, $(g \circ f)^{-1}(x)$, the inverse function of $g \circ f(x)$

(3 marks)

Inverse is $x = \sqrt{1 - \dfrac{9}{y^2}}$

$x^2 = 1 - \dfrac{9}{y^2}$

$\dfrac{9}{y^2} = 1 - x^2$

$y^2 = \dfrac{9}{1-x^2}$

$\therefore (g \circ f)^{-1}(x) = \sqrt{\dfrac{9}{1-x^2}}$

✓ interchange x, y
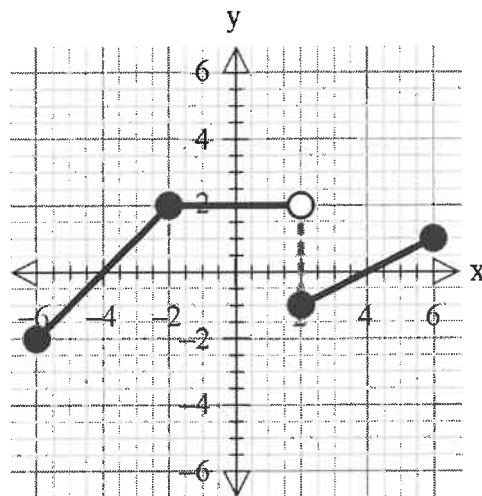
✓ algebraic manipulation

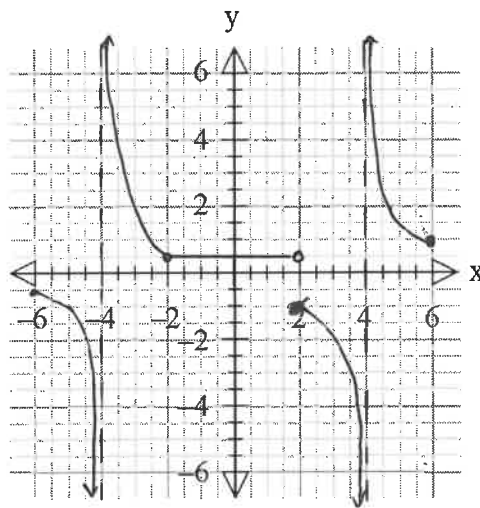(Is never -ve)

✓ states answer

2

⑨

Question 2 (5 marks)

Consider the graph of the function $y = h(x)$ shown below.

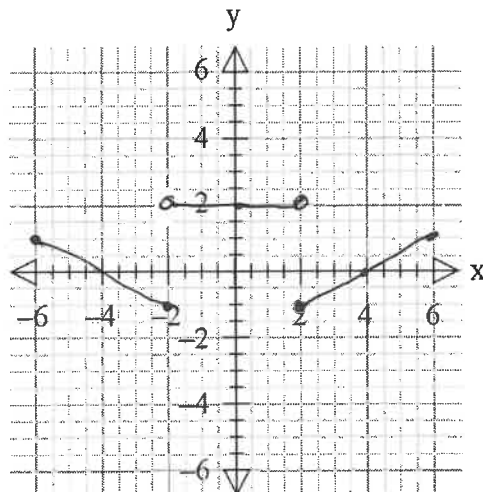(a) On the axes provided draw a graph of $y = \dfrac{1}{h(x)}$. (3 marks)

✓ vertical asymptotes at $x = \pm 4$

✓ $y = \frac{1}{2}$ $\quad -2 \le x < 2$

✓ approaches asymptotes from correct sides

(b) On the axes provided draw a graph of $y = h(|x|)$. (2 marks)

✓ same as $y = h(x)$ for $x \ge 0$

✓ symmetry in y-axis

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⑤

Question 3 (6 marks)

Consider the function $g(x) = \dfrac{x^2 + 1}{x - 2}$.

(a) State the equation of the vertical asymptote for the graph of $y = g(x)$. (1 mark)

$$x = 2$$

✓ answer

(b) Show algebraically, that $g(x)$ can be written in the form $ax + b + \dfrac{c}{x-2}$, stating clearly the values of $a, b,$ and c. (3 marks)

$$\frac{x^2 + 1}{x - 2} = ax + b + \frac{c}{x - 2}$$

$$x^2 + 1 = (ax + b)(x - 2) + c$$

✓ multiplies by $x - 2$

$$\therefore \quad x^2 : \quad 1 = a$$
$$x : \quad 0 = b - 2a$$
$$1 : \quad 1 = -2b + c$$

$$\frac{x^2 + 1}{x - 2} = x + 2 + \frac{5}{x - 2}$$

✓ compares coefficients

$$\therefore \quad \underline{a = 1} \;,\; \underline{b = 2} \;,\; \underline{c = 5}$$

✓ states a, b, c

(c) Explain clearly what writing the function in the form $g(x) = ax + b + \dfrac{c}{x-2}$ indicates about the graph of the function. (2 marks)

As $x \to \pm\infty$ the graph approaches the line $y = x + 2$ (an oblique asymptote)

✓ behaviour as $x \to \pm\infty$

✓ correct terminology

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⑥

Question 4 (8 marks)

Consider the functions $p(x) = k|(x-6)(x+2)|$ and $q(x) = a - 6|x|$

Given that $y = p(x)$ and $y = q(x)$ meet when $x = 1$ and $x = 6$.

(a) Find the values of k and a. (3 marks)

$x = 1:$ $\quad k \times 5 \times 3 = a - 6$

$x = 6:$ $\quad 0 = a - 36$

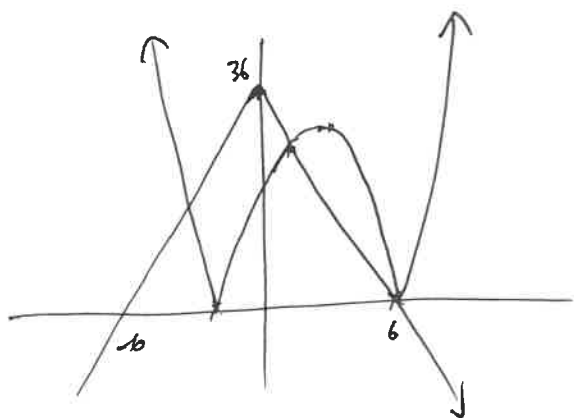$\therefore a = 36$

$15k = 36 - 6$

$k = 2$

✓ uses $x = 1$ to find $15k = a - 6$

✓ uses $x = 6$ to find $a = 36$

✓ answers

(b) Find the x coordinate of any other points of intersection of the two graphs. (3 marks)

Meet again when

$36 + 6x = 2(x-6)(x+2)$ ✓ equation

$36 + 6x = 2x^2 - 8x - 24$

$0 = x^2 - 7x - 30$

$0 = (x - 10)(x + 3)$ ✓ factorises quadratic

Since $x < 0$ (from graph)

Meet again at $(-3, 18)$ ✓ point

(c) State the range of values of b for which the equation $p(x) = b$ has exactly 4 solutions. (2 marks)

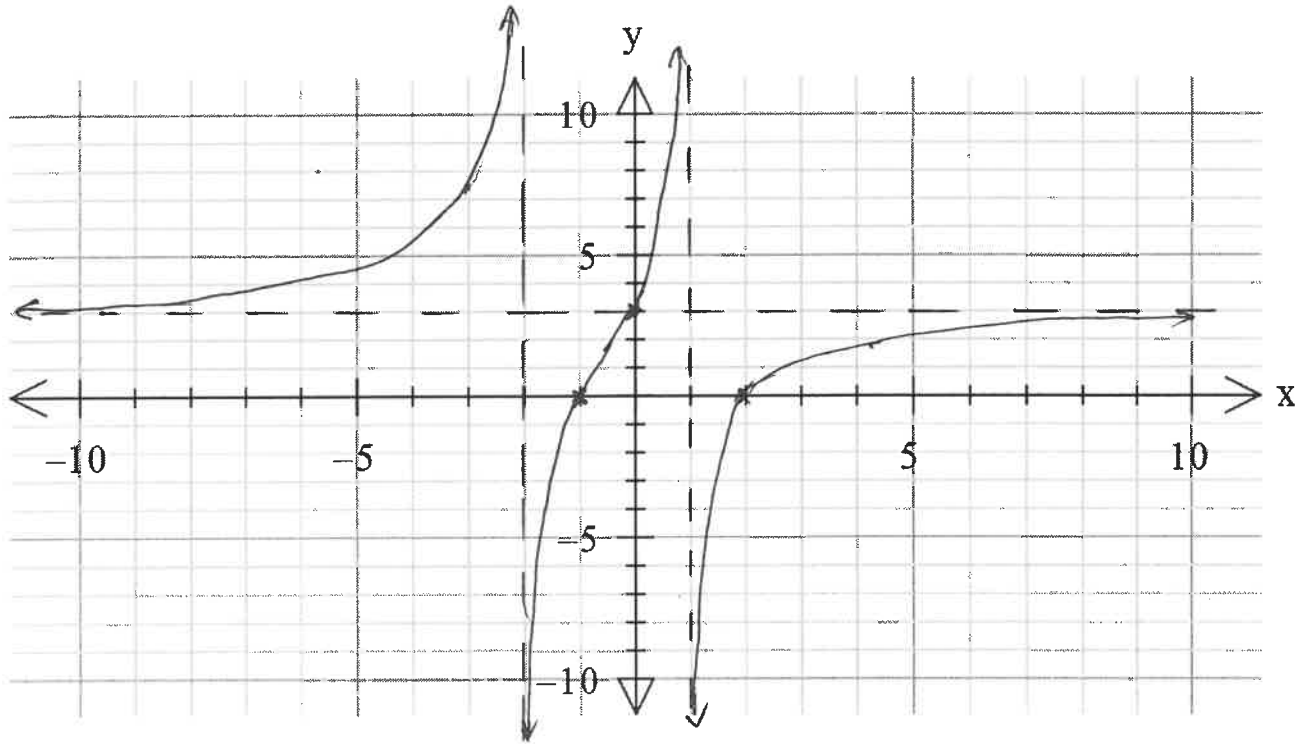Max value when $x = 2$

$f(2) = 32$

$\therefore 0 < b < 32$

✓ finds $f(2) = 32$

✓ correct range of values

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On the axes below draw the graph of $f(x) = \dfrac{3(x-2)(x+1)}{(x+2)(x-1)}$, showing all features.

$\checkmark$ — vertical asymptotes at $x = -2$, $x = 1$

$\checkmark$ — x intercepts at $x = -1$, $x = 2$

$\checkmark$ — y intercept

$\checkmark$ — horizontal asymptote $y = 3$

$\checkmark$ — approaches all asymptotes correctly

$\boxed{5}$

Question 6 (4 marks)

Consider the function $g(x) = \dfrac{2}{x+2}$.

If $f \circ g(x) = \dfrac{x+8}{x+2}$, find $f(x)$

$$f(u) = \frac{x+8}{x+2} \quad \text{where} \quad u = \frac{2}{x+2}$$

$$\therefore \quad x+2 = \frac{2}{u}$$

$$x = \frac{2}{u} - 2$$

✓ writes x in terms of u

$$\therefore \quad f(u) = \frac{\frac{2}{u} - 2 + 8}{\frac{2}{u} - \cancel{2} + \cancel{2}}$$

✓ substitutes for x in $f(u)$

$$= \frac{\frac{2}{u} + 6}{\frac{2}{u}}$$

$$= \frac{2 + 6u}{2}$$

✓ algebraic manipulation

$$= 1 + 3u$$

$$\therefore \quad \underline{\underline{f(x) = 3x + 1}}$$

✓ states $f(x)$

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